

ALGORITMI I STRUKTURE PODATAKA 2



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ACN K1-23

① FIND-PEAK(a, n)

$a[0] = -\infty$, $a[n+1] = -\infty$

low = 1; high = n

while (low $\leq$ high) do

mid = (low + high) / 2

if ($a[\text{mid}] > a[\text{mid}-1]$ and $a[\text{mid}] > a[\text{mid}+1]$) then

return mid

end-if

if ($a[\text{mid}] > a[\text{mid}-1]$ and $a[\text{mid}] < a[\text{mid}+1]$) then

low = mid + 1

else high = mid - 1

end-if

end-while

② BST-SUM(root, sum)

STACK-INIT(Slow), STACK-INIT(Shigh)

tmp = root

while (tmp != nil) do PUSH(Slow, tmp), tmp = left(tmp)

tmp = root

while (tmp != nil) do PUSH(Shigh, tmp), tmp = right(tmp)

low = POP(Slow), high = POP(Shigh)

while (key(low) < key(high)) do

if (key(low) + key(high) = sum) then return low, high

if (key(low) + key(high) < sum) then

if (right(low) != nil) then ... low = ...

else low = POP(Slow)

else

if (left(high) != nil) then ... high = ...

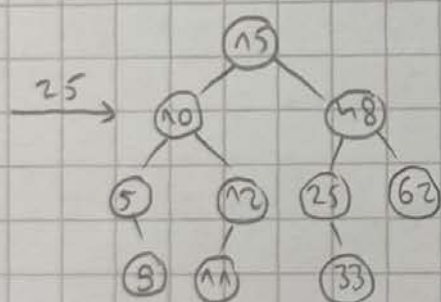
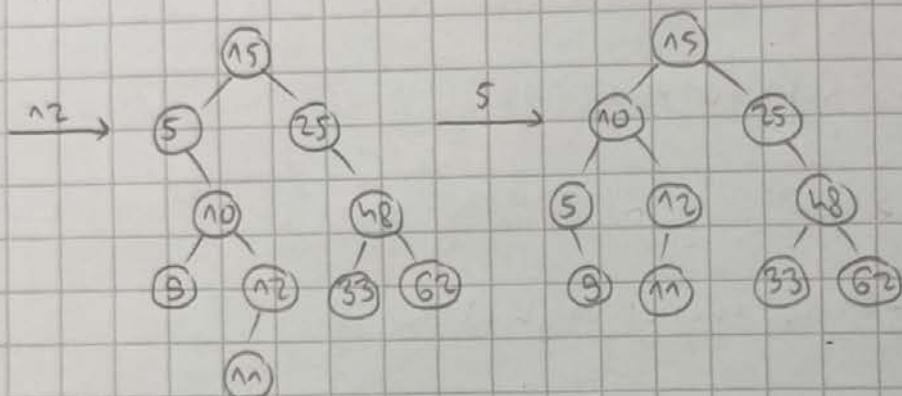
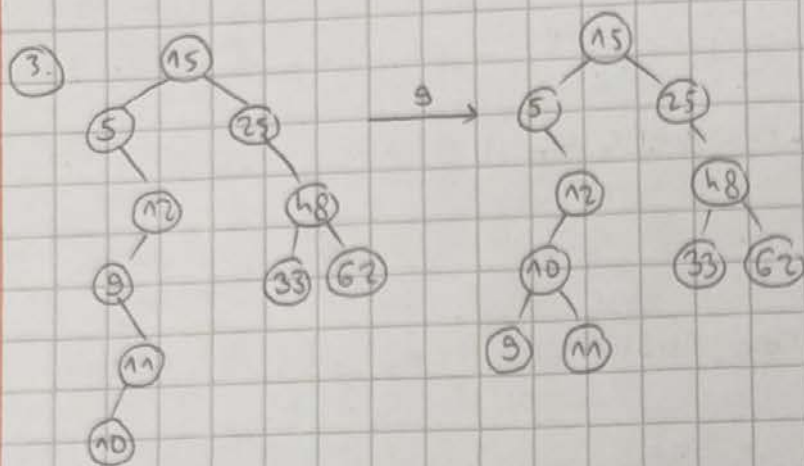
else high = POP(Shigh)

end-if

end-while

return nil, nil

ACN Kn-23



④ 1 5 7 16 19 21 34 39 49 54 55 61 63 76 89 90 93 96 99

- претрета на 9 49 63 91 96

9 $\rightarrow$ 5 поретјена $\Rightarrow 0$

49 $\rightarrow$ 7 поретјена $\Rightarrow 9$

63 $\rightarrow$ 6 поретјена $\Rightarrow 13$

91 $\rightarrow$ 6 поретјена $\Rightarrow 0$

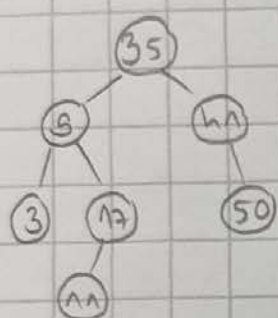
96 $\rightarrow$ 3 поретјена $\Rightarrow 18$

- резултат : [0, 9, 13, 0, 18]

- укупан број поретјена : 27

⑤ - ако чвор има лево подстабло $\rightarrow$ претходник
је највећи чвор у левом подстаблу

- ако чвор нема лево подстабло $\rightarrow$ претходник
је највиши чвор који је постапран
чвор у десном подстаблу

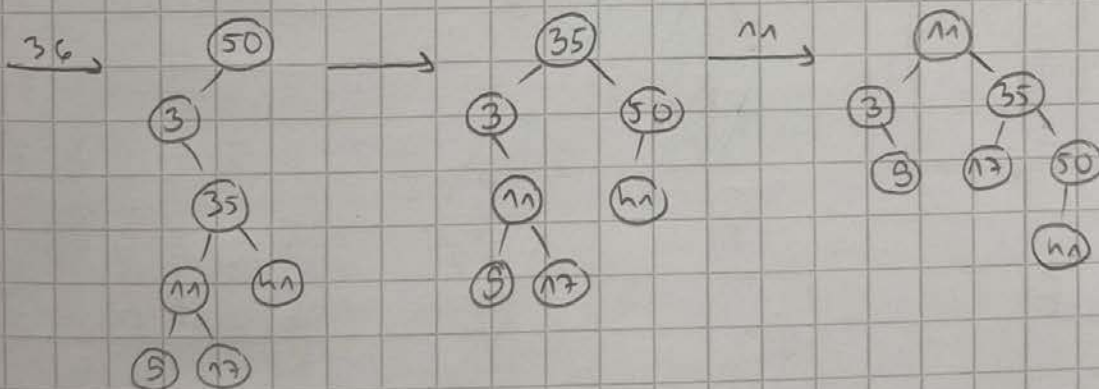
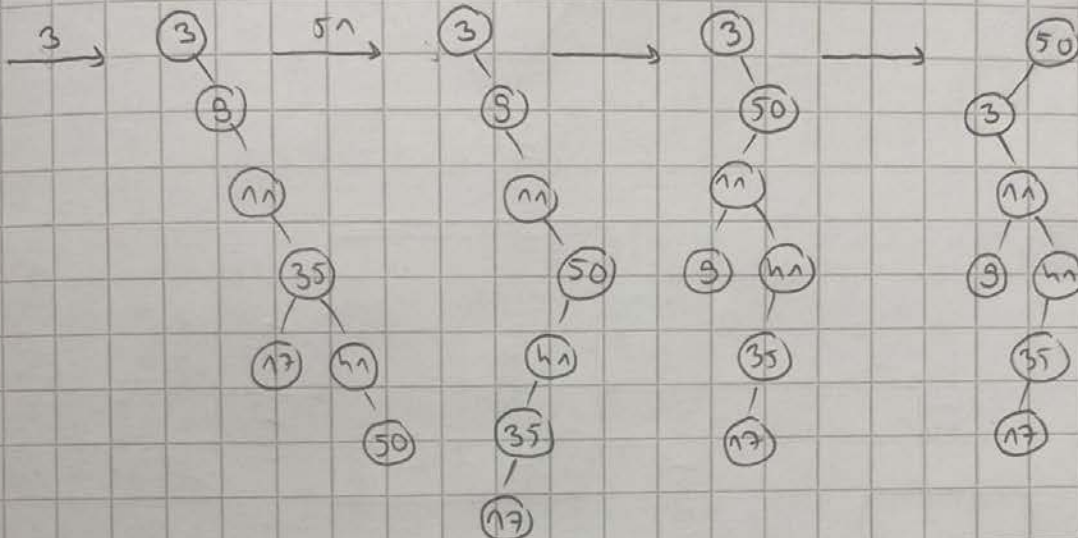
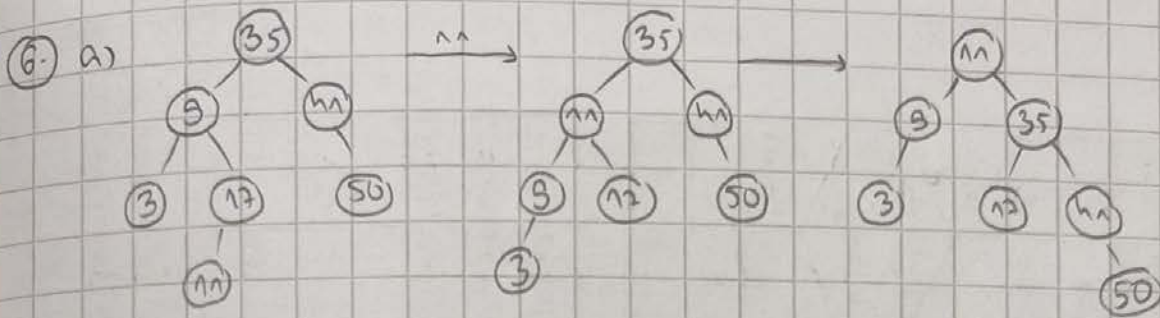


35 претходник: има лево подстабло $\Rightarrow$ 17

11 претходник: нема лево подстабло $\Rightarrow$ 9

УЛ-23

ACD



б) самоподемаляйте стадо:

$$h + 3 + 6 + 5 + 3 = 21 \text{ коран}$$

однако BST стадо:

$$h + 3 + 3 + 2 + h = 16 \text{ корана}$$

① - претрета на 4, 8, 16, 16, 8, 4

- транспозиција: $\Rightarrow \frac{34}{6} = 5,67$ поређења

14	12	8	4	11	2	9	10	7	16	22	4
14	12	4	8	11	2	9	10	7	16	22	4
14	12	8	4	11	2	9	10	7	16	22	10
14	12	8	4	11	2	9	10	16	7	22	9
14	12	8	4	11	2	9	16	10	7	22	3
14	8	12	4	11	2	9	16	10	7	22	4
14	8	4	12	11	2	9	16	10	7	22	

- пребацивање на почетак: $\Rightarrow \frac{24}{6} = 4$ поређења

14	12	8	4	11	2	9	10	7	16	22	4
4	14	12	8	11	2	9	10	7	16	22	4
8	4	14	12	11	2	9	10	7	16	22	10
16	8	4	14	12	11	2	9	10	7	22	1
16	8	4	14	12	11	2	9	10	7	22	2
8	16	4	14	12	11	2	9	10	7	22	3
4	8	16	14	12	11	2	9	10	7	22	

② RECONSTRUCT_BST (preorder, n)

root = GETNODE, info(root) = preorder[0]

parent(root) = nil, tmp = root

for (i = 1 to n-1) do

new = GETNODE, info(new) = preorder[i]

if (info(new) < info(tmp) then

left(tmp) = new, parent(new) = tmp

else

while (tmp != root) do

if (tmp = left(parent(tmp)) and

info(tmp) < info(new) < info(parent(tmp)) then break

tmp = parent(tmp)

end-while

while (right(tmp) != nil) do tmp = right(tmp)

right(tmp) = new, parent(new) = tmp

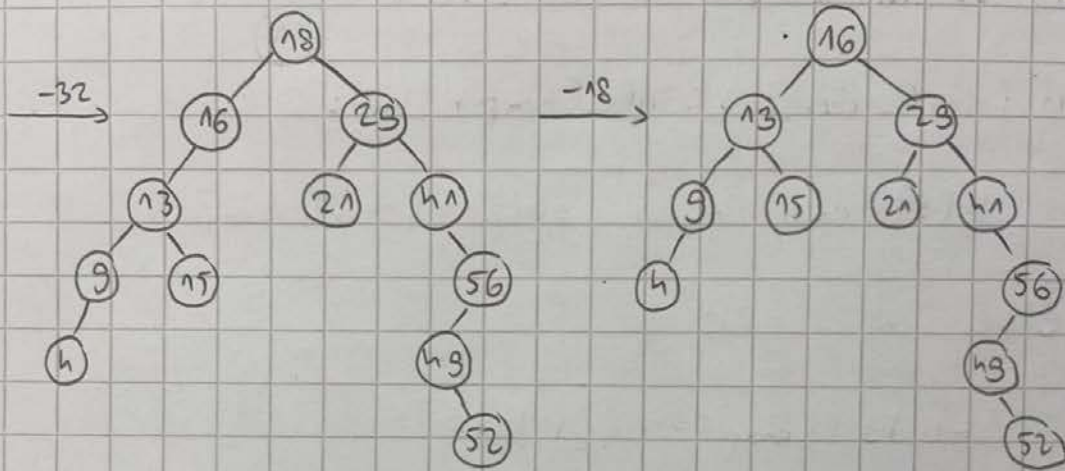
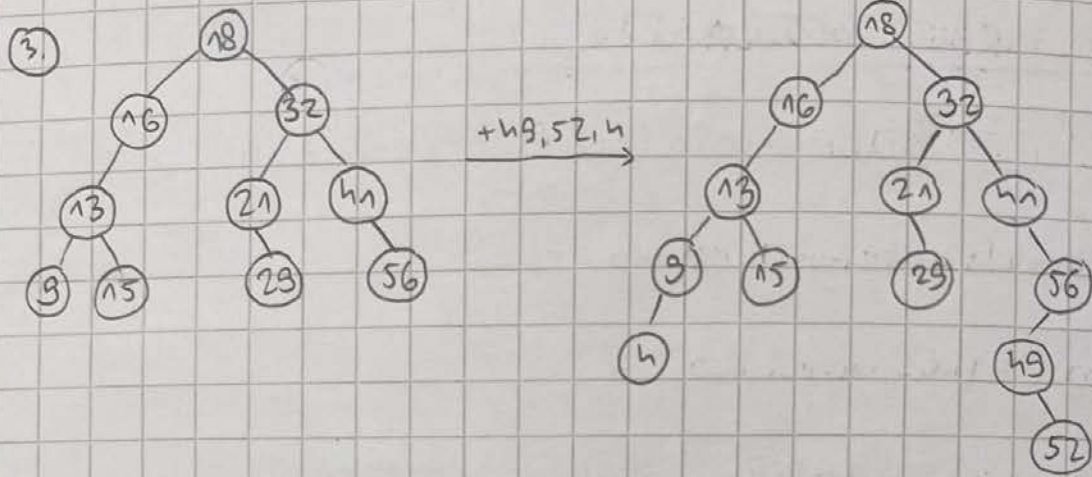
end-if

tmp = new

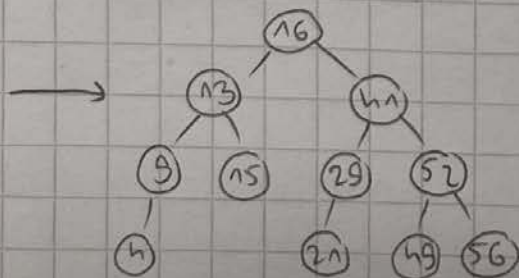
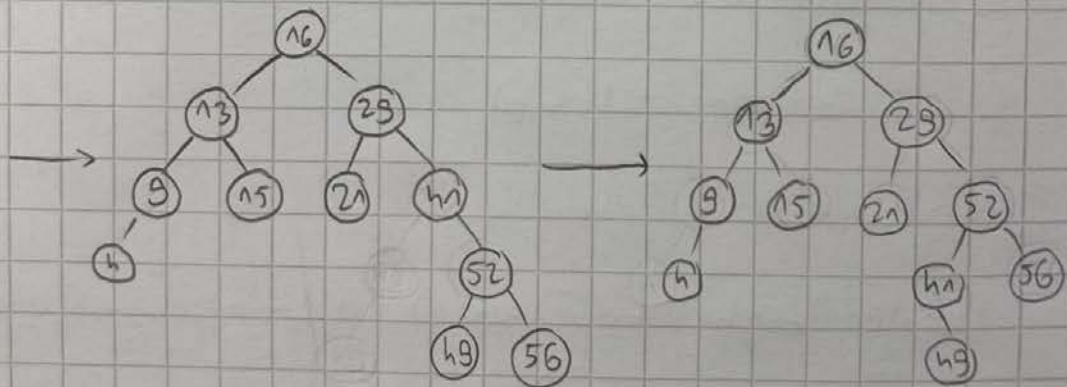
end-for

return root

ACM 11-22



-уборка са левим далаком: 56, 41, 29, 16



④ FIND-FIRST-AND-LAST(a, n, k)

low = 0, high = n - 1

while (low ≤ high) do

mid = (low + high) / 2

if ($a[mid] = k$) then break

else if ($a[mid] > k$) then high = mid - 1

else low = mid + 1

end-while

if ($a[mid] \neq k$) return -1, -1

first = mid, last = mid

while (first > 0) do

if ($a[first - 1] = k$) then first = first - 1

end-while

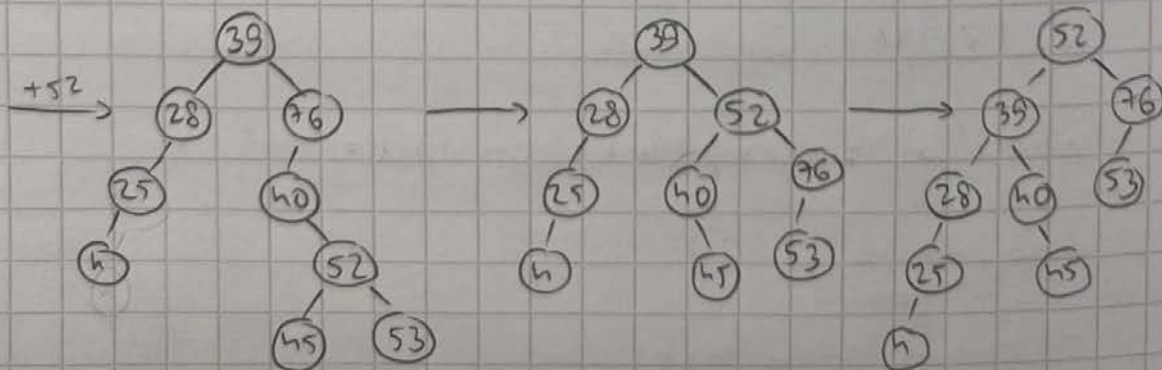
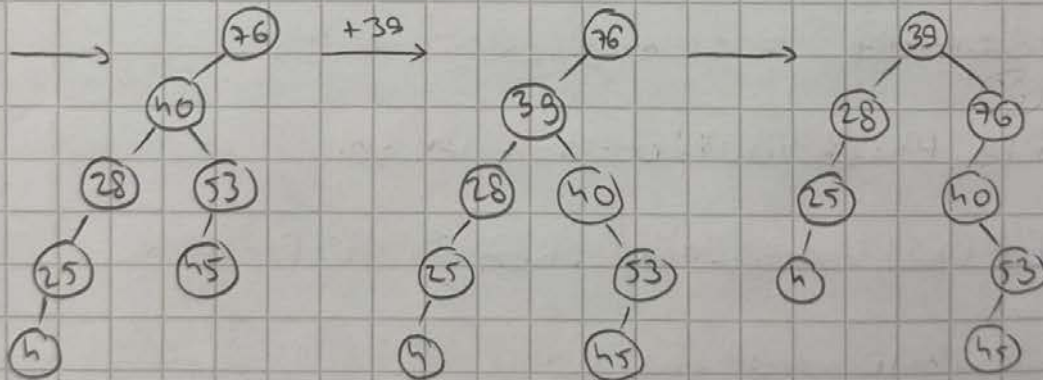
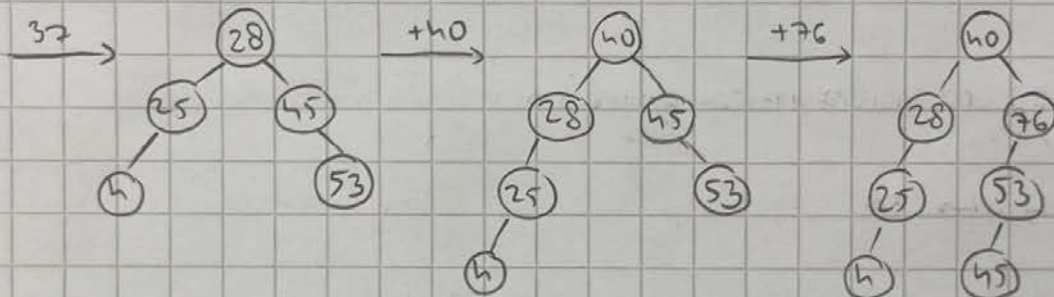
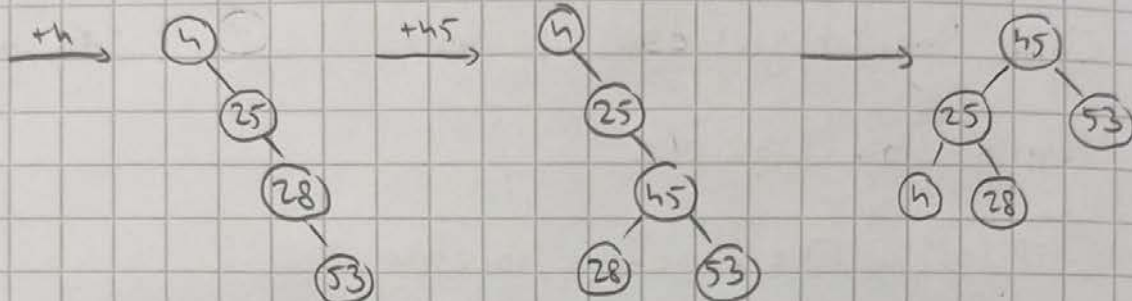
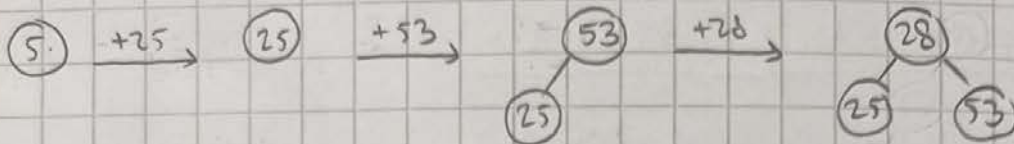
while (last < n - 1) do

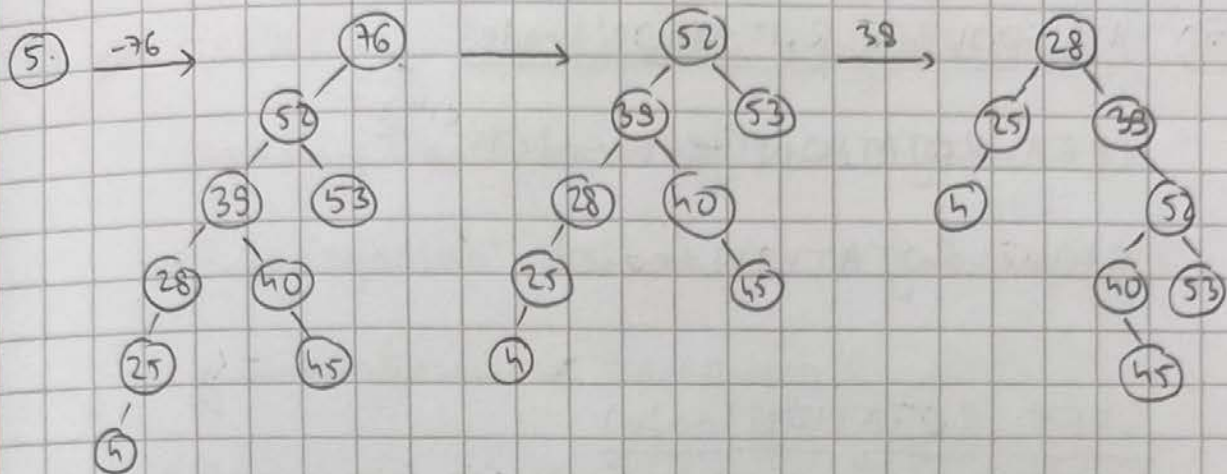
if ($a[last + 1] = k$) then last = last + 1

end-while

return first, last

ACN K1-22





⑥ $P(x) = -4x^2 + 12x + 7$

- може се одредити интервал неизвесности, тј. интервал за који важи да су вредности полинома у горњој и доњој граници супротан знак, па онда применити бинарно претраживање

$$P(0) = 7, P(1) = 15, P(2) = 15, P(4) = -9$$

$$\Rightarrow \text{интервал неизвесности } (2, 4)$$

$$P(3) = 7 \Rightarrow \text{интервал } (3, 4)$$

$$P(3.5) = 0 \quad \checkmark$$

- дакле, нула полинома је $x = 3.5$

ACN KA-22

⑦ AVL-DOUBLE-ROTATION (node)

LEFT-ROTATION (left(node))

RIGHT-ROTATION (node)

LEFT-ROTATION (node)

tmp = right(node)

right(node) = left(tmp)

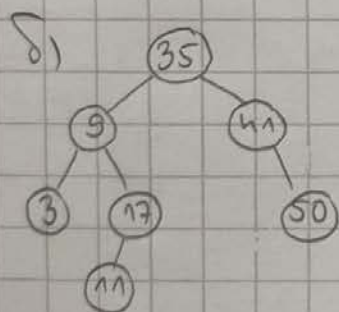
left(tmp) = node

parent(tmp) = parent(node)

parent(node) = tmp

KL-22

- 8) а) - претпоставља да сви кључеви имају једнаку вероватноћу присуства и да је „случајно“ BST настало уметанњем у произвољном поретку
- закључак да је просечна перформанса уметрањивања у „случајном“ BST-у истог реда $O(\log n)$ као у оптималном BST-у и постоји за константан фактор $n^{\log 2}$.

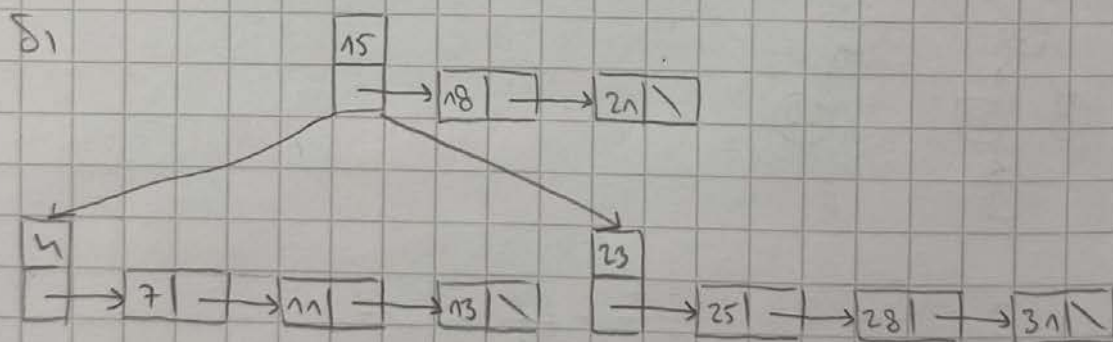


$$S_n = \frac{P_1 + n}{n} = \frac{11 + 7}{7} = \frac{18}{7} \approx 2,57$$

$$U_n = \frac{PE}{n+1} = \frac{P_1 + 2n}{n+1} = \frac{11 + 2 \cdot 7}{8} = \frac{25}{8} = 3,125$$

Асп кл-21

- ① а) - користити се BST, чији чвор има поље
попуна са адресом прве инструкције
мене функције, и друго поље са
показивачем на исту другу инструкцију
које припадају тој функцији
- при претрази се налази остварајућа
функција, тј. адреса мене прве
инструкције, а затим пролази кроз
исту другу инструкцију



- Претражује се чвор 23 у BST
- Пролази се кроз исту чвора са чвором
23 до адресе 25

$\sqrt{n-2}$

ACN

② CALC-SHOP-NUM(price, n, x)

if (price[0] > x) return 0

if (price[n-1] < x) return n

low = 0, high = n-1

while (low < high) do

mid = (low + high) / 2

if (price[mid] > x and price[mid-1] < x) then

return n - mid

end-if

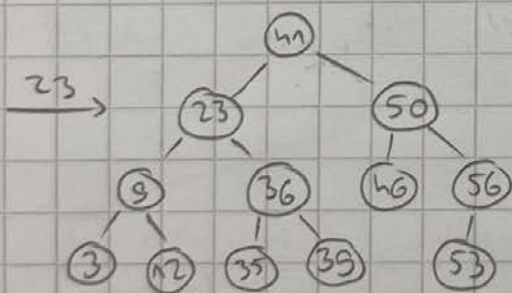
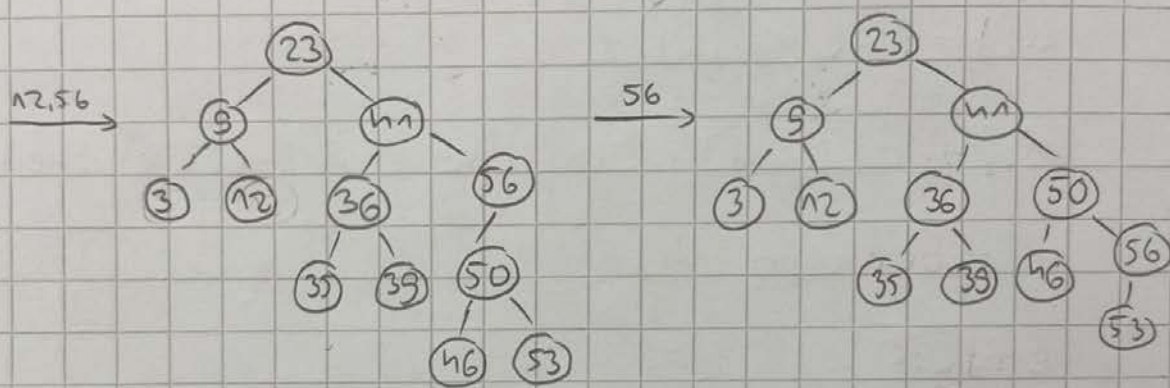
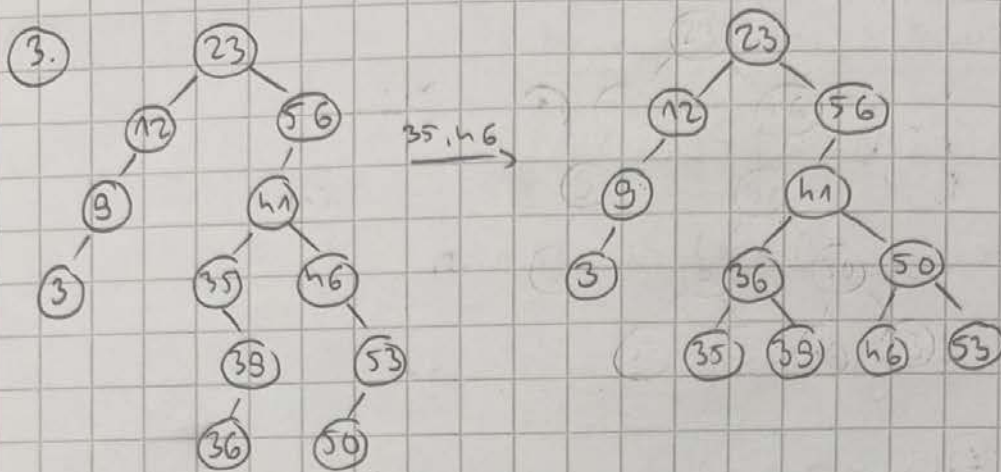
if (price[mid] > x) then high = mid - 1

else low = mid + 1

end-if

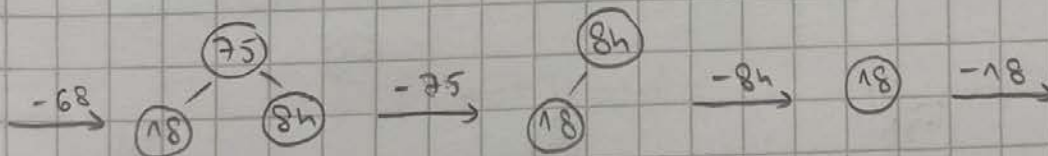
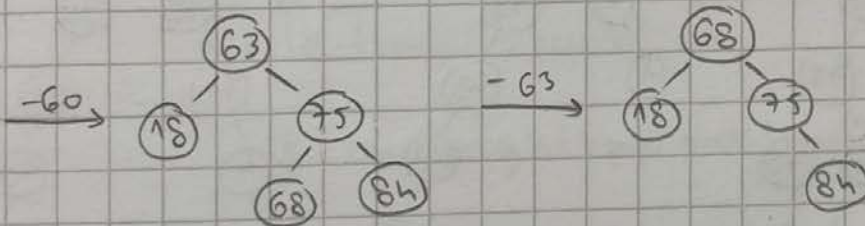
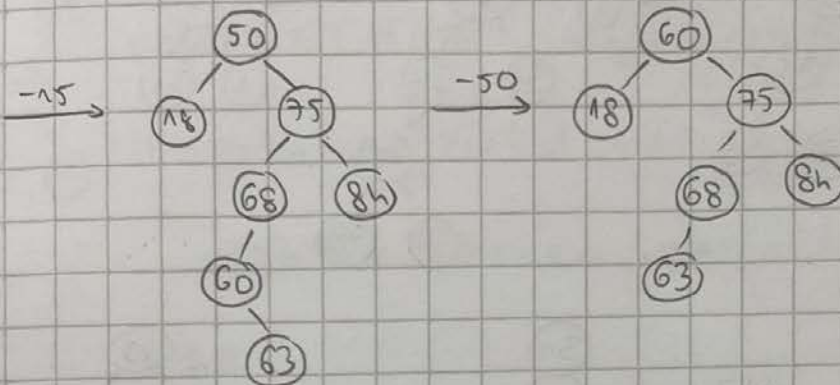
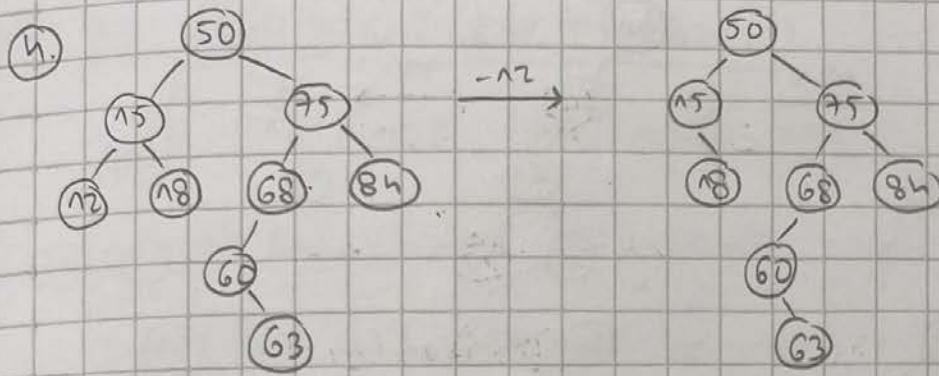
end-while

ACN 121-21

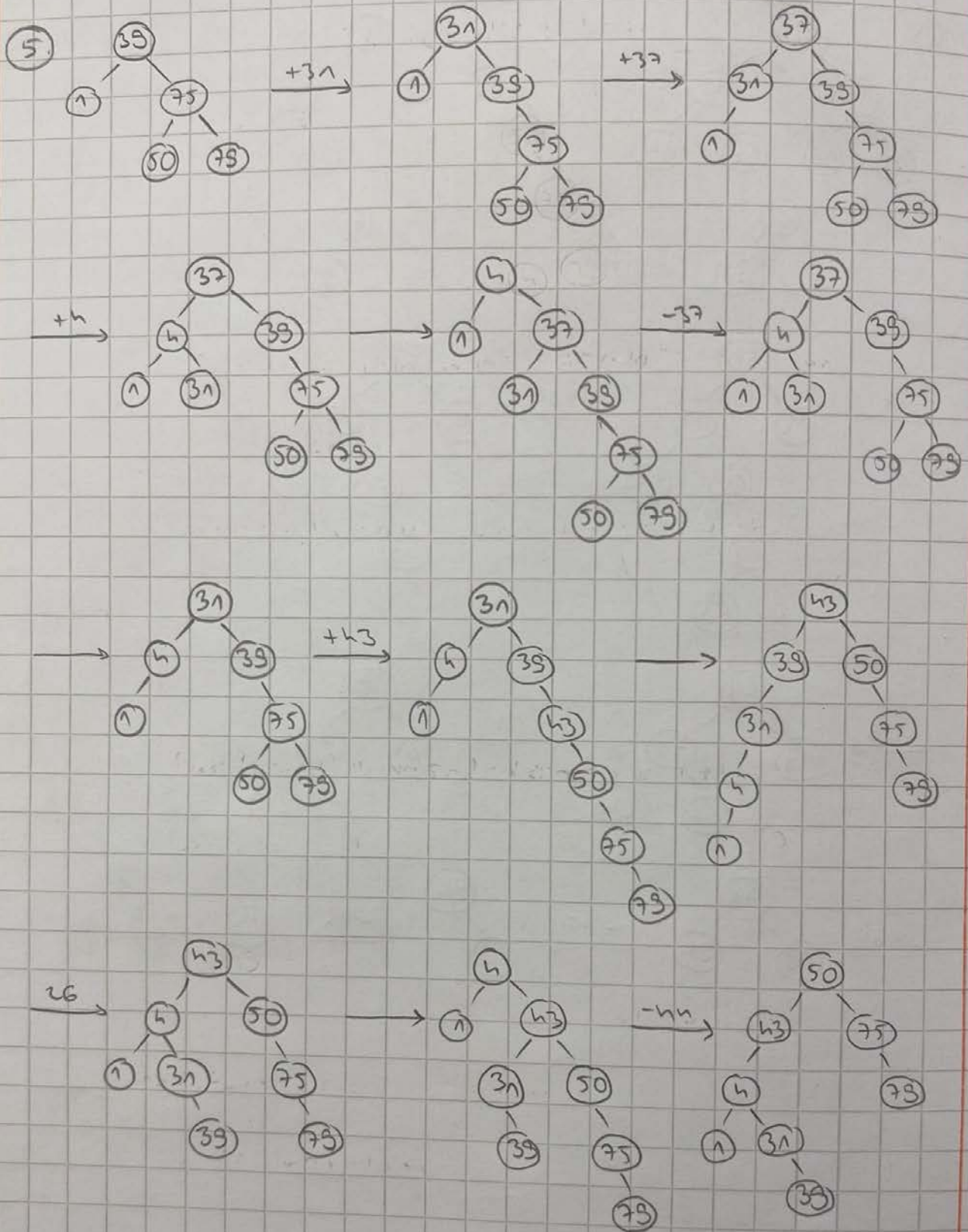


K1-21

Acn



ACD K1-21



⑥ BST- RECONSTRUCT(post, n)

root = GET NODE, info(root) = post[n]

parent(root) = nil, tmp = root

for (i = n-2 down to 0) do

new = GET NODE, info(new) = post[i]

if (info(new) > info(tmp) then

right(tmp) = new, parent(new) = tmp

else

while (tmp != root) do

if (tmp = right(parent(tmp)) and

info(tmp) < info(new) < info(parent(tmp)) then break

tmp = parent(tmp)

end-while

while (left(tmp) != nil) do tmp = left(tmp)

left(tmp) = new, parent(new) = tmp

end-if

tmp = new

end-for

return root

ACN 41-21

⑦ AVL-FIND-CRITICAL-NODE(node)

change = true

critical = nil

tmp = node

while (change and parent(tmp) $\neq$ nil) do

 p = parent(tmp)

 if (tmp = left(p)) then balance(p) = balance(p) + 1

 else balance(p) = balance(p) - 1

 end-if

 if (critical = nil and abs(balance(p)) > 1) then

 critical = p

 end-if

 if (balance(p) = 0) then change = false

 else tmp = parent(tmp)

 end-if

end-while

return critical

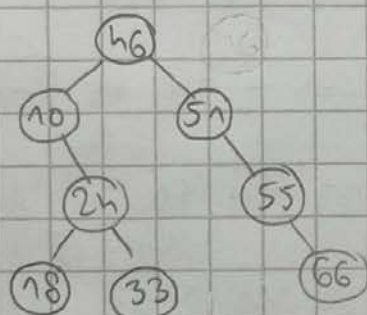
8. а) - уметанье нбучелв по отадајућим
вероватноћама p_i

- цена стабна: $C = \sum_{i=1}^n p_i (h_i + 1)$

$\delta) h_i$:	10	18	24	33	46	51	55	66
p_i :	0.15	0.02	0.12	0.03	0.3	0.25	0.08	0.05

- редом се уметну нбучелви

46, 51, 10, 24, 55, 66, 33, 18



$$\Rightarrow C = 0.3 \cdot 1 + (0.25 + 0.15) \cdot 2 + (0.12 + 0.08) \cdot 3 + \\ + (0.05 + 0.03 + 0.02) \cdot 4$$

$$C = 0.3 + 0.8 + 0.6 + 0.4 = 2.1$$

АСП КЛ-20

① - претрата на 25, 40, 25, 40

- транспозиција: $\Rightarrow \frac{28}{n} = 7$ поретјења

12 7 3 27 25 78 98 21 22 40 51

5

12 7 3 25 27 78 98 21 22 40 51

10

12 7 3 25 27 78 98 21 40 22 51

4

12 7 25 3 27 78 98 21 40 22 51

9

12 7 25 3 27 78 98 40 21 22 51

- пребацување на почетак: $\Rightarrow \frac{19}{n} = 4,75$ поретјења

12 7 3 27 25 78 98 21 22 40 51

5

25 12 7 3 27 78 98 21 22 40 51

10

40 25 12 7 3 27 78 98 21 22 51

2

25 40 12 7 3 27 78 98 21 22 51

2

40 25 12 7 3 27 78 98 21 22 51

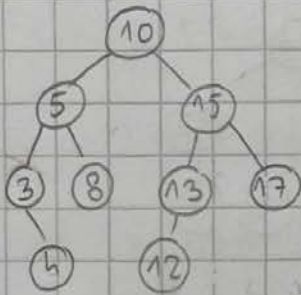
KL-20

predhodnik

② - претходник по inorder сортирање за
задати чвор

1) ако има лево подстаблo → највећи чвор
у левом подстаблu

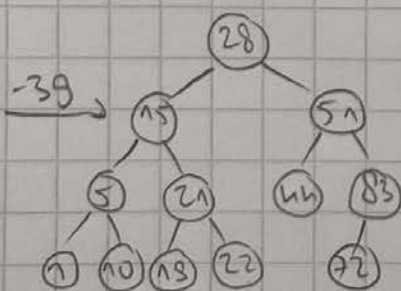
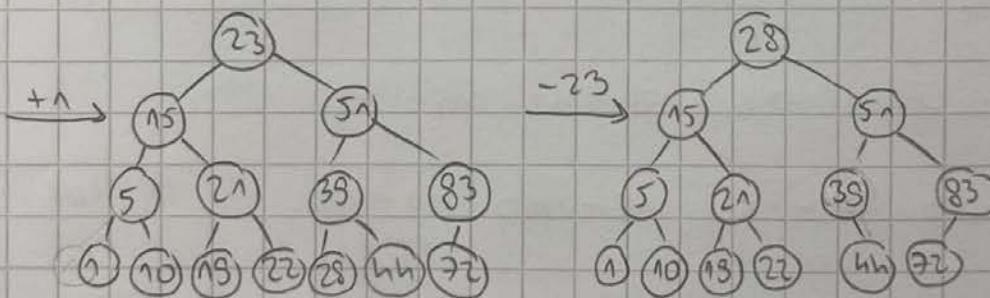
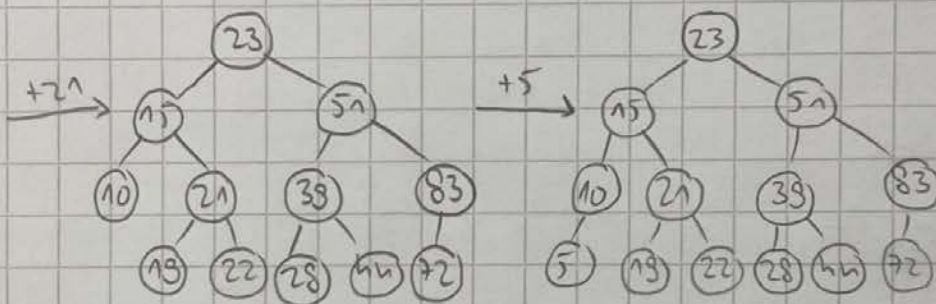
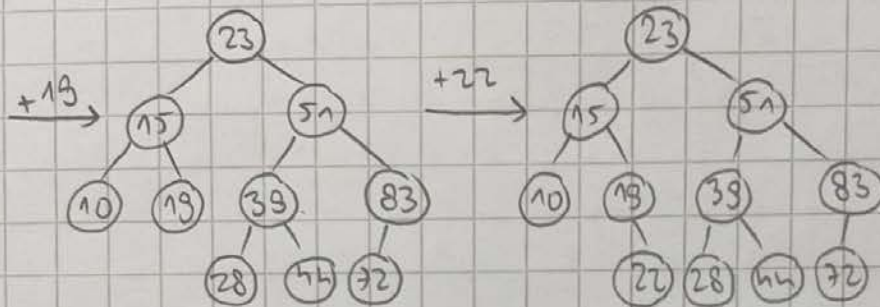
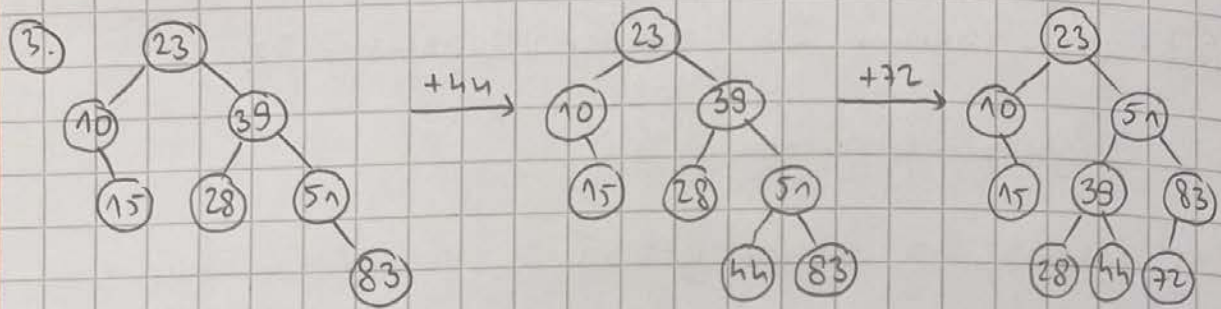
2) ако нема лево подстаблo → најближи
предак коме је десни чвор у
десном подстаблu



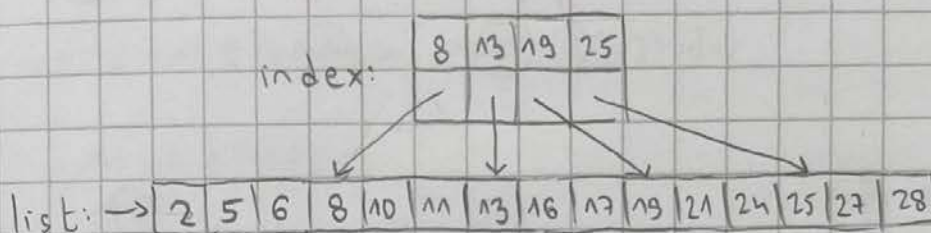
1.) $5 \rightarrow 4$, $10 \rightarrow 8$, $13 \rightarrow 12$, $15 \rightarrow 13$

2.) $11 \rightarrow 3$, $8 \rightarrow 5$, $12 \rightarrow 10$, $17 \rightarrow 15$

ACN 11-20



в) а) - индекс содержит мажи број елиминационите
 кључева из листе и показување на те
 кључеве у дамој листи



б) FIND-KEY-LIST(list, index, size, key)

tmp = list, i = 0

while (key > info(index[i]) and i < size) do

tmp = adr(index[i])

i = i + 1

end-while

while (tmp != nil) do

if (info(tmp) = key) then return tmp

else if (info(tmp) > key) then return nil

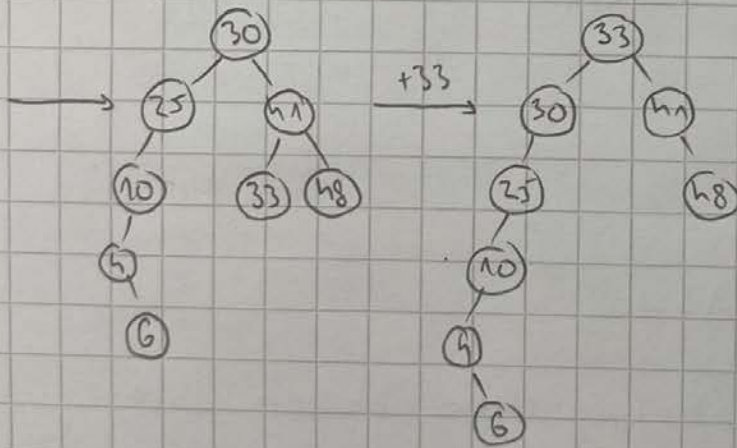
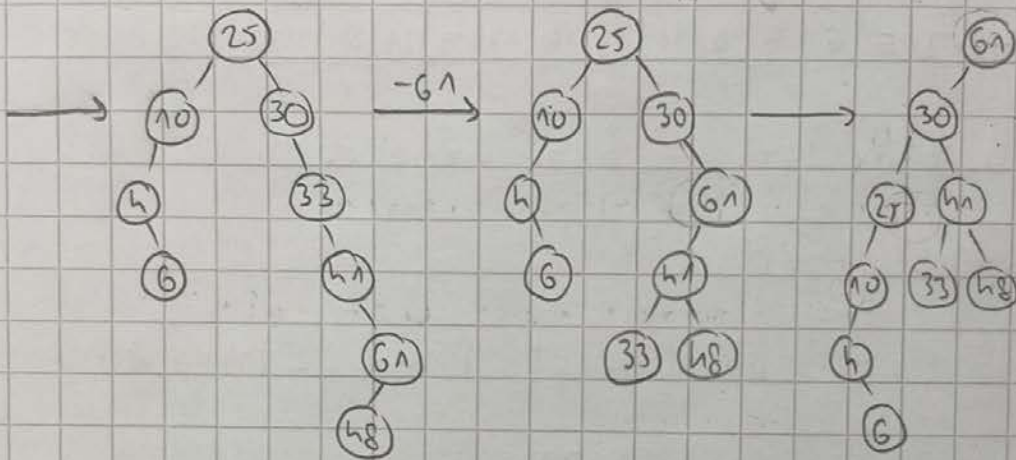
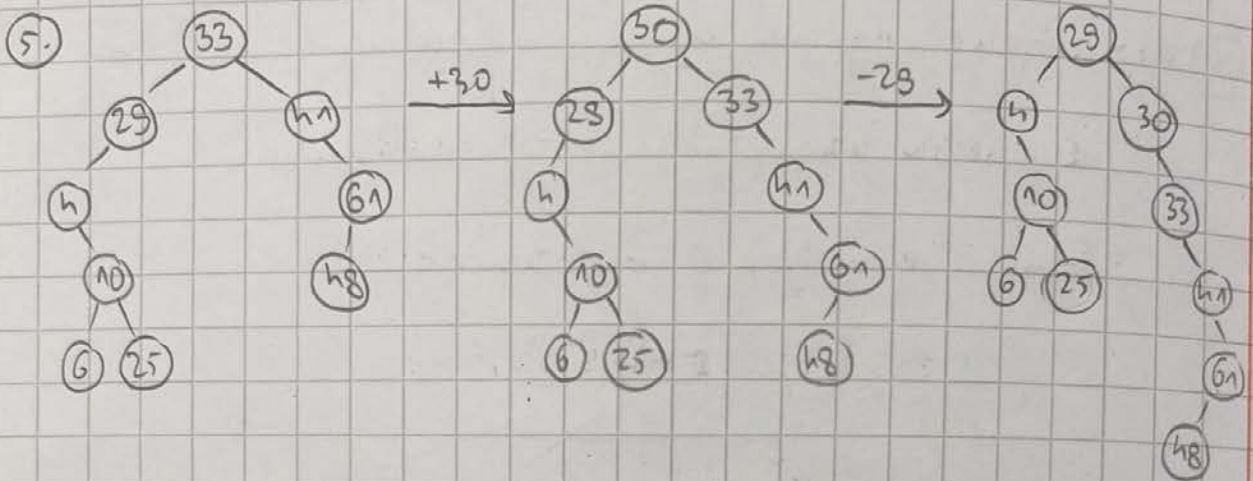
end-if

tmp = next(tmp)

end-while

return nil

ACM 11-20



⑥ PRINT_ALL_BST (root)

$n = 0$, preorder = NEWARRAY (NODE)

STACK_INIT (S), PUSH (S, root)

while (not STACK_EMPTY (S)) do

 next = POP (S)

 while (next != nil) do

 preorder [n] = next, $n = n + 1$

 if (right (next) != nil) then PUSH (S, right (next))

 next = left (next)

 end-while

end-while

for (i = n - 1 down to 0) do

 p = preorder [i]

 if ((left (p) = nil or (bst (left (p)) and info (left (p)) < info (p))) and

 (right (p) = nil or (bst (right (p)) and info (right (p)) > info (p)))) then

 bst (p) = true, print (p)

 else

 bst (p) = false

 end-if

ACN K1-20

② FIND-FIRST_POSITIVE(f)

low = 0, high = 1

while (f(high) < 0) do

low = high, high = high * 2

end-while

while (low < high) do

mid = low + (high - low) * (-f(low)) / (f(high) - f(low))

if (f(mid) = 0) return mid

if (f(mid) > 0) then high = mid

else low = mid

end-if

end-while

- прво одређујемо интервал неизвесности, а онда та методом интерполације

преграђивања смањујемо, тако да је вредност функције у његовим крајевима супротан знака

$$(8) a) S_n = 1 + (U_0 + U_1 + \dots + U_{n-1})/n$$

$$S_n = \frac{p(1+n)}{n}$$

$$U_n = \frac{PE}{n+n}$$

$$d) PE = p(1+2n)$$

$$S_n = \frac{PE - 2n + n}{n} = \frac{PE - n}{n} = \frac{n+1}{n} U_n - 1$$

ACD KL-13

1) - уменителе 5, 28, 29, укладателе 14, 15

h	2	10	14	15	15	21	24	25	40	45
h	0	0	1	0	0	0	1	0	1	0

↓ +5

h	5	5	14	15	15	21	24	25	40	45
h	1	1	0	1	0	0	1	0	1	0

↓ +28

h	5	5	14	15	15	21	24	25	28	40
h	1	1	0	1	0	0	1	0	1	1

↓ +29

h	5	5	14	15	15	21	24	28	29	40
h	1	1	0	1	0	0	1	1	1	1

↓ -14

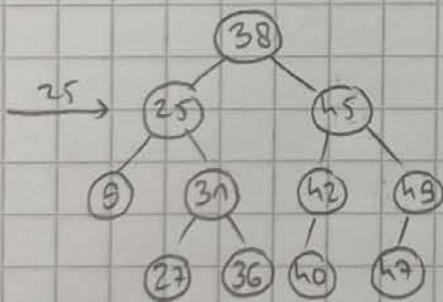
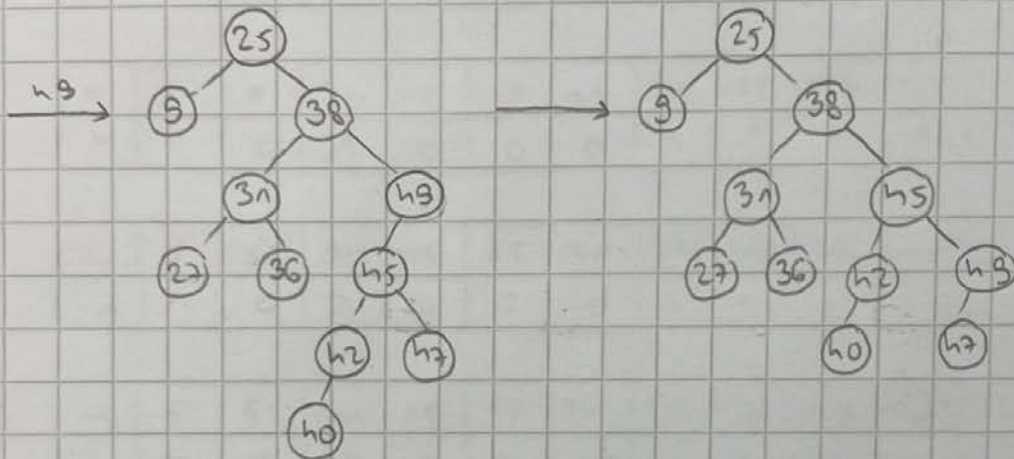
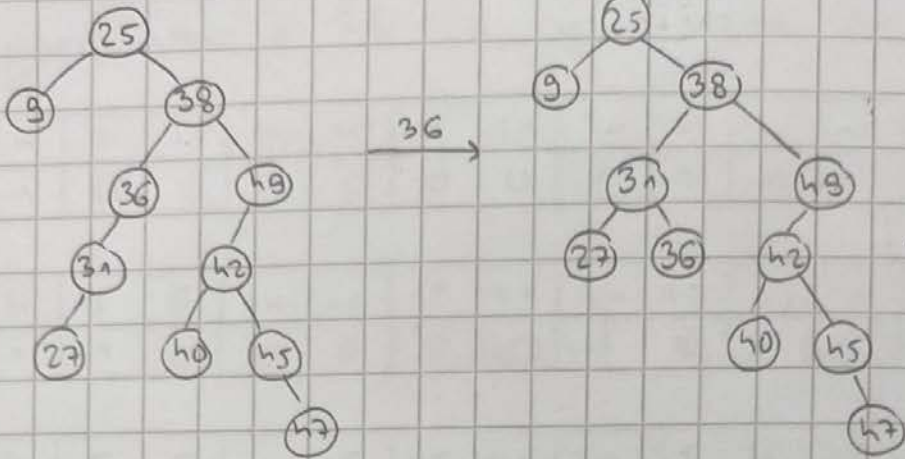
h	5	5	14	15	15	21	24	28	29	40
h	1	1	0	0	0	0	1	1	1	1

↓ -15

h	5	5	14	15	15	21	24	28	29	40
h	1	1	0	0	0	0	1	1	1	1

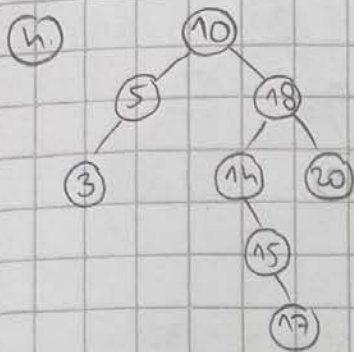
ACD W1-19

(3.)



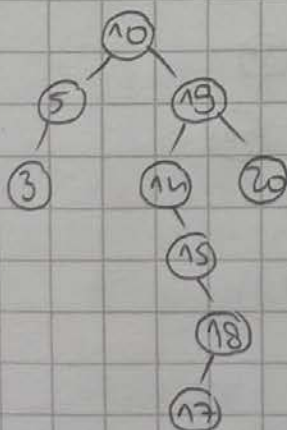
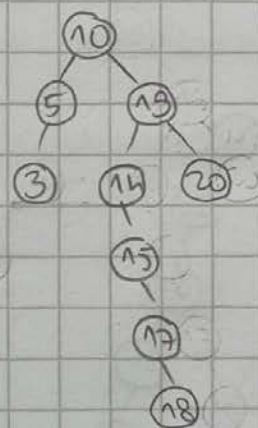
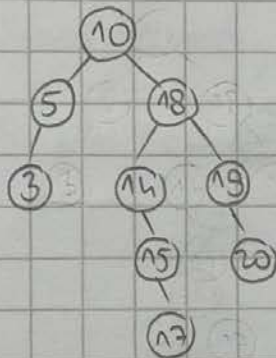
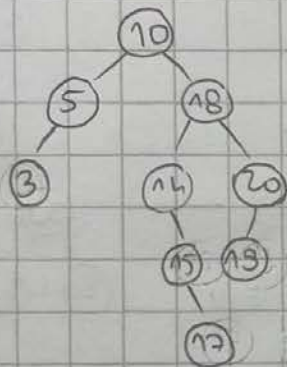
14.1-19

ACD

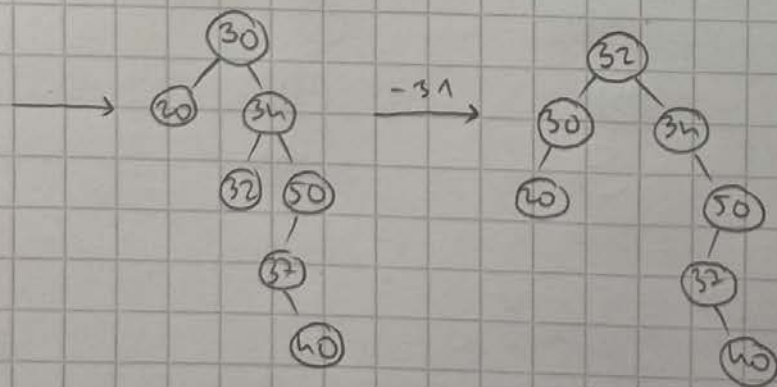
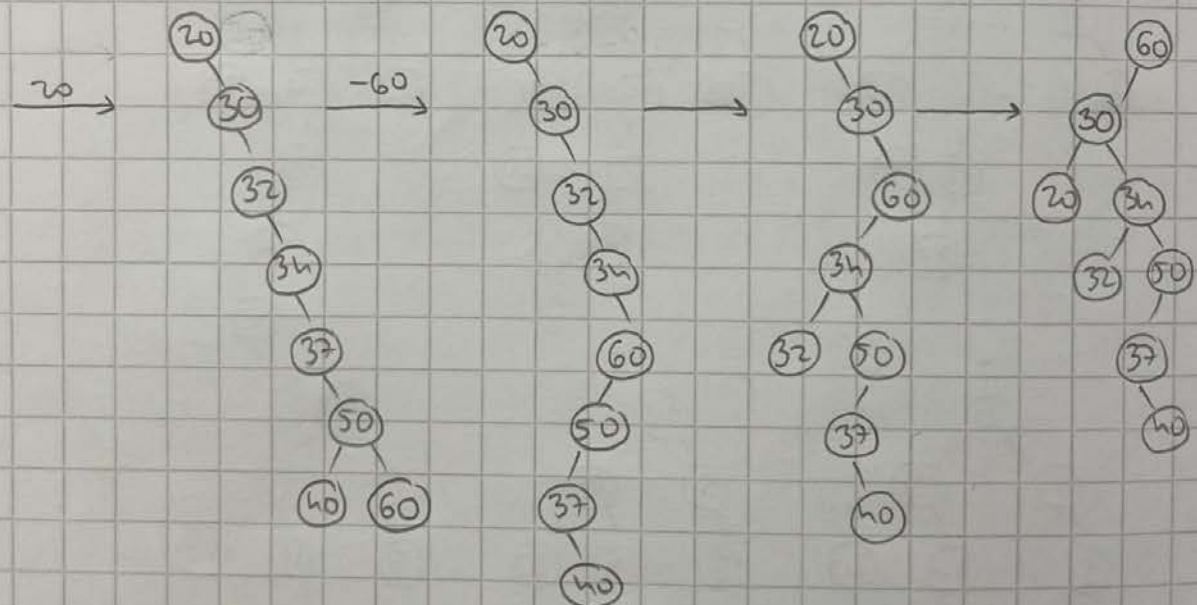
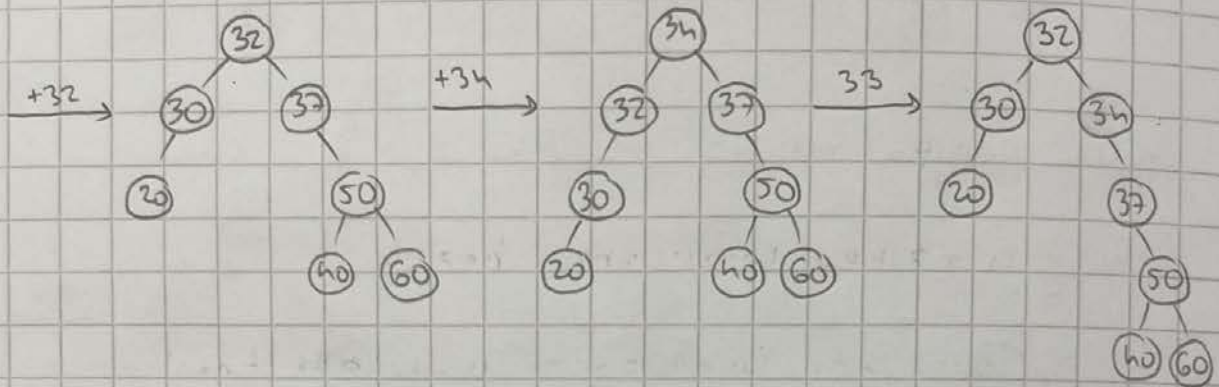
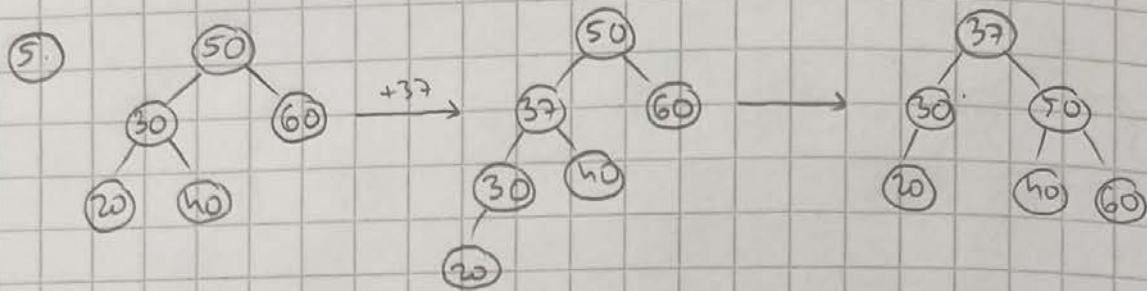


- два можат да излезат една пре друга

клуча 19:



ACD WA-19



⑥ SPLAY_SPLIT(root, k)

tmp = root

p = nil

while (tmp != nil) do

 p = tmp

 if (k = key(tmp)) then break

 if (k < key(tmp)) then tmp = left(tmp)

 else tmp = right(tmp)

 end-if

end-while

if (tmp = nil)

 SPLAY(p)

 if (k > key(p)) return p, right(p)

 else return left(p), p

 end-if

else

 SPLAY(tmp)

 return left(tmp), right(tmp)

end-if

② FIND-BST-FLOOR(root, key)

tmp = root

p = nil

while (tmp != nil) do

 if (k = key(tmp)) then return tmp

 if (k > key(tmp)) then tmp = right(tmp)

 p = tmp

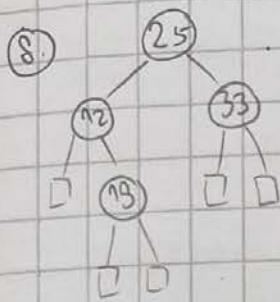
 tmp = left(tmp)

else tmp = left(tmp)

end-if

end-while

return p



h_i	12	19	25	33
p_i	0.2	0.2	0.15	0.05

h_i	$h_i + 12$	$12 + h_i + 19$	$19 + h_i + 25$	$25 + h_i + 33$	$33 + h_i$
q_i	0.05	0.1	0.15	0.05	0.05

а) цена сграда: $C = \sum_{i=1}^n p_i(h_i + 1) + \sum_{i=0}^n q_i h_i$

$$C = 0.2 \cdot 2 + 0.2 \cdot 3 + 0.15 + 0.05 \cdot 2 + 0.05 \cdot 2 + 0.1 \cdot 3 + 0.15 \cdot 3 + 0.05 \cdot 2 + 0.05 \cdot 2 = 0.4 + 0.6 + 0.15 + 0.1 + 0.1 + 0.3 + 0.1 + 0.1 = 2.3$$

б) корен 19 $\rightarrow w_L = q_0 + p_1 + q_1 = 0.05 + 0.2 + 0.1 = 0.35$

$$w_r = q_2 + p_3 + q_3 + p_4 + q_4 = 0.15 + 0.15 + 0.05 + 0.05 + 0.05 = 0.45$$

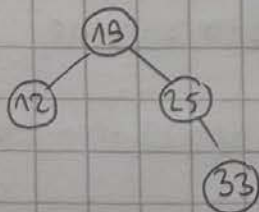
$$|w_L - w_r| = 0.1$$

корен 25 $\rightarrow w_L = q_0 + p_1 + q_1 + p_2 + q_2 = 0.05 + 0.2 + 0.1 + 0.2 + 0.15 = 0.7$

$$w_r = q_3 + p_4 + q_4 = 0.05 + 0.05 + 0.05 = 0.15$$

$$|w_L - w_r| = 0.55$$

$\Rightarrow$ за корен се бира 19, а за сини 25



а) $C = \sum_{i=1}^n p_i(k_{i+1}) + \sum_{i=0}^n q_i \cdot k_i$

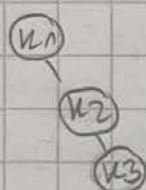
k_i - висина интернот / екстернот нора

p_i - вероятността успешните претрете на нора k_i

q_i - вероятността неуспешните претрете на нора k_i у интервалу (k_i, k_{i+1})

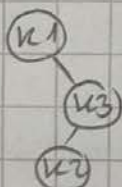
б) $k_1 < k_2 < k_3$, $p_1 = 0.2$, $p_2 = 0.05$, $p_3 = 0.1$

$q_0 = 0.15$, $q_1 = 0.2$, $q_2 = 0.1$, $q_3 = 0.2$



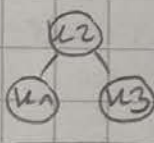
$$C = 0.2 + 0.05 \cdot 2 + 0.1 \cdot 3 + 0.15 + 0.2 \cdot 2 + (0.1 + 0.2) \cdot 3 =$$

$$= 0.2 + 0.1 + 0.3 + 0.15 + 0.4 + 0.9 = 2.05$$



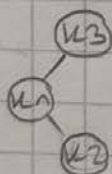
$$C = 0.2 + 0.1 \cdot 2 + 0.05 \cdot 3 + 0.15 + (0.2 + 0.1) \cdot 3 + 0.2 \cdot 2 =$$

$$= 0.2 + 0.2 + 0.15 + 0.15 + 0.9 + 0.4 = 2$$



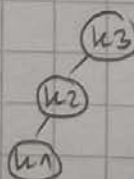
$$C = 0.05 + (0.2 + 0.1) \cdot 2 + (0.15 + 0.2 + 0.1 + 0.2) \cdot 2 =$$

$$= 0.05 + 0.6 + 1.3 = \underline{1.95}$$



$$C = 0.1 + 0.2 \cdot 2 + 0.05 \cdot 3 + 0.15 \cdot 2 + (0.2 + 0.1) \cdot 3 + 0.2 =$$

$$= 0.1 + 0.4 + 0.15 + 0.3 + 0.9 + 0.2 = 2.05$$



$$C = 0.1 + 0.05 \cdot 2 + 0.2 \cdot 3 + (0.15 + 0.2) \cdot 3 + 0.1 \cdot 2 + 0.2 =$$

$$= 0.1 + 0.1 + 0.6 + 1.05 + 0.2 + 0.2 = 2.25$$

② FIND-BST-SUCC(root, key)

tmp = root

p = nil

while (tmp != nil) do

if (key = key(tmp)) then break

if (key < key(tmp)) then

p = tmp

tmp = left(tmp)

else tmp = right(tmp)

end-if

end-while

if (right(tmp) != nil) then

tmp = right(tmp)

while (left(tmp) != nil) do tmp = left(tmp)

return tmp

else return p

end-if

④ CORRECT-BST(root)

$n=0$, $next=root$, $STACK_INIT(s)$, $wrong=nil$

while (true) do

 while ($next \neq nil$) do

$PUSH(s, next)$, $next = left(next)$

 end-while

 if (not $STACK_EMPTY(s)$) then

$next = POP(s)$

$a[n] = next$, $n = n + 1$

$next = right(next)$

 else break

 end-if

end-while

for ($i=0$ to $n-1$) do

 if ($(a[i] > a[i-1] \text{ and } a[i] > a[i+1])$ or

$(a[i] < a[i-1] \text{ and } a[i] < a[i+1])$) then

 if ($wrong = nil$) $wrong = a[i]$

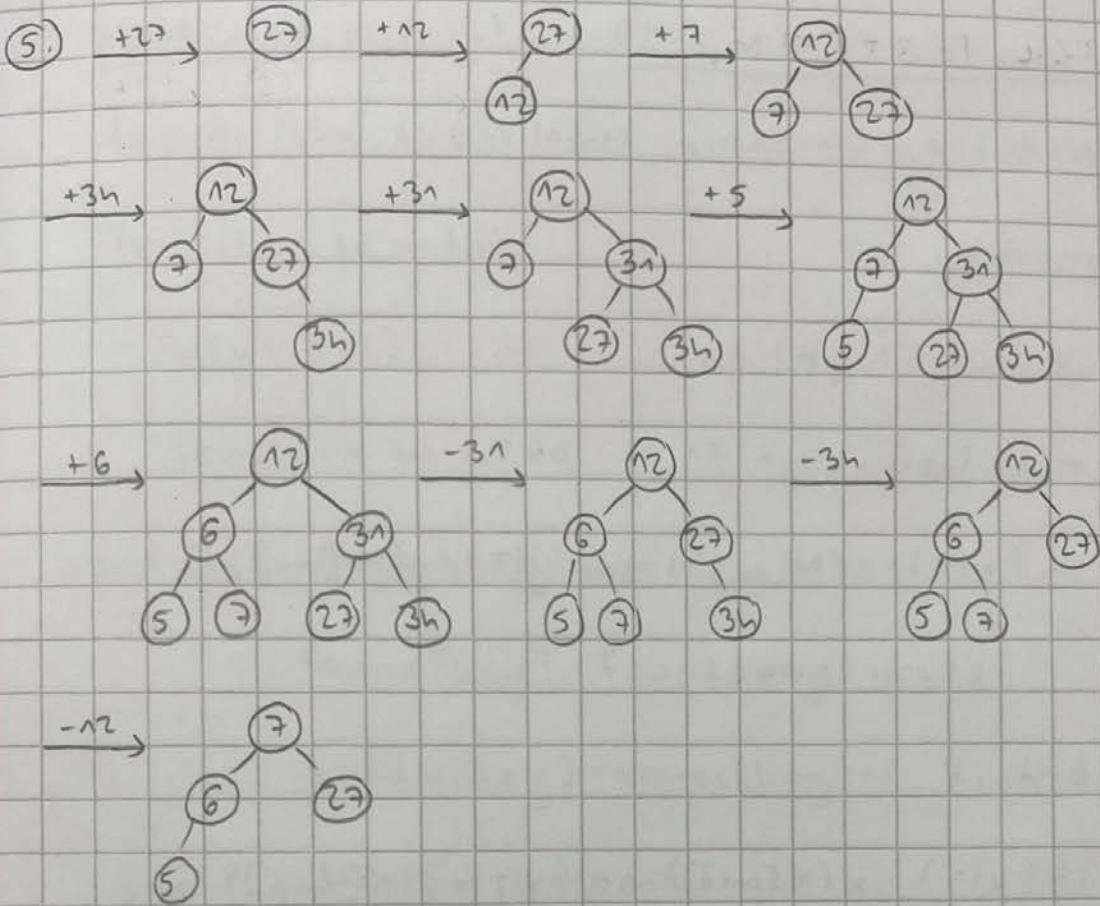
 else $SWITCH(wrong, a[i])$

 end-if

end-for

12.1-18

Acn



① CHECK_VERTEX(M, n, T)

low = 0

high = $n-1$

while (low $\leq$ high) do

mid = (low + high) / 2

if ($x(T) = x(M[mid])$ and $y(T) = y(M[mid])$) then

return true

end-if

if ($x(T) > x(M[mid])$ or ($x(T) = x(M[mid])$ and

$y(T) > y(M[mid])$)) then low = mid + 1

else high = mid - 1

end-if

end-while

return false

⑦ SEARCH(A, k, n, m)

found = NEW_ARRAY(100), times = NEW_ARRAY(100)

for (i = 1 to m) do

key = k[i]

for (j = 1 to n) do

if (A[j] = key) then

found[key] = found[key] + j

times[key] = times[key] + 1

MOVE(j, times[key])

break

end-if

end-for

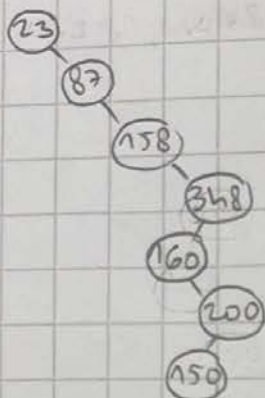
end-for

avg = found / times

return avg

ACM K1-18

g) a) $23 \rightarrow 87 \rightarrow 158 \rightarrow 348 \rightarrow 160 \rightarrow 200 \rightarrow 150 \rightarrow 158$

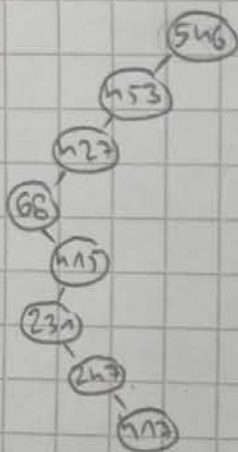


nije validna sekvencija

jer 150 ne može biti u

desnom podskupu 160

g) b) $546 \rightarrow 453 \rightarrow 422 \rightarrow 68 \rightarrow 415 \rightarrow 231 \rightarrow 243 \rightarrow 412 \rightarrow 300$



nije validna sekvencija

jer 112 ne može biti u

levom podskupu 115

①

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
3	7	10	11	18	22	25	27	39	41	45	48	56	64	65	78	86

- Бинарна претрата на 25 и 41:

$$4 \text{ поређења за } 25 + 4 \text{ поређења за } 41 = 8$$

$\Rightarrow 8$ поређења

- Вишеструка секвенцијална претрата на 25 и 41:

$$8 \text{ поређења за } 25 + 6 \text{ поређења за } 41$$

$\Rightarrow 14$ поређења

④ ARR-PRED-SUCC(arr, k)

low = 1, high = n

while (low ≤ high) do

mid = (low + high) / 2

if (arr[mid] = k) then

return mid - 1, mid + 1

end-if

if (k > arr[mid]) then low = mid + 1

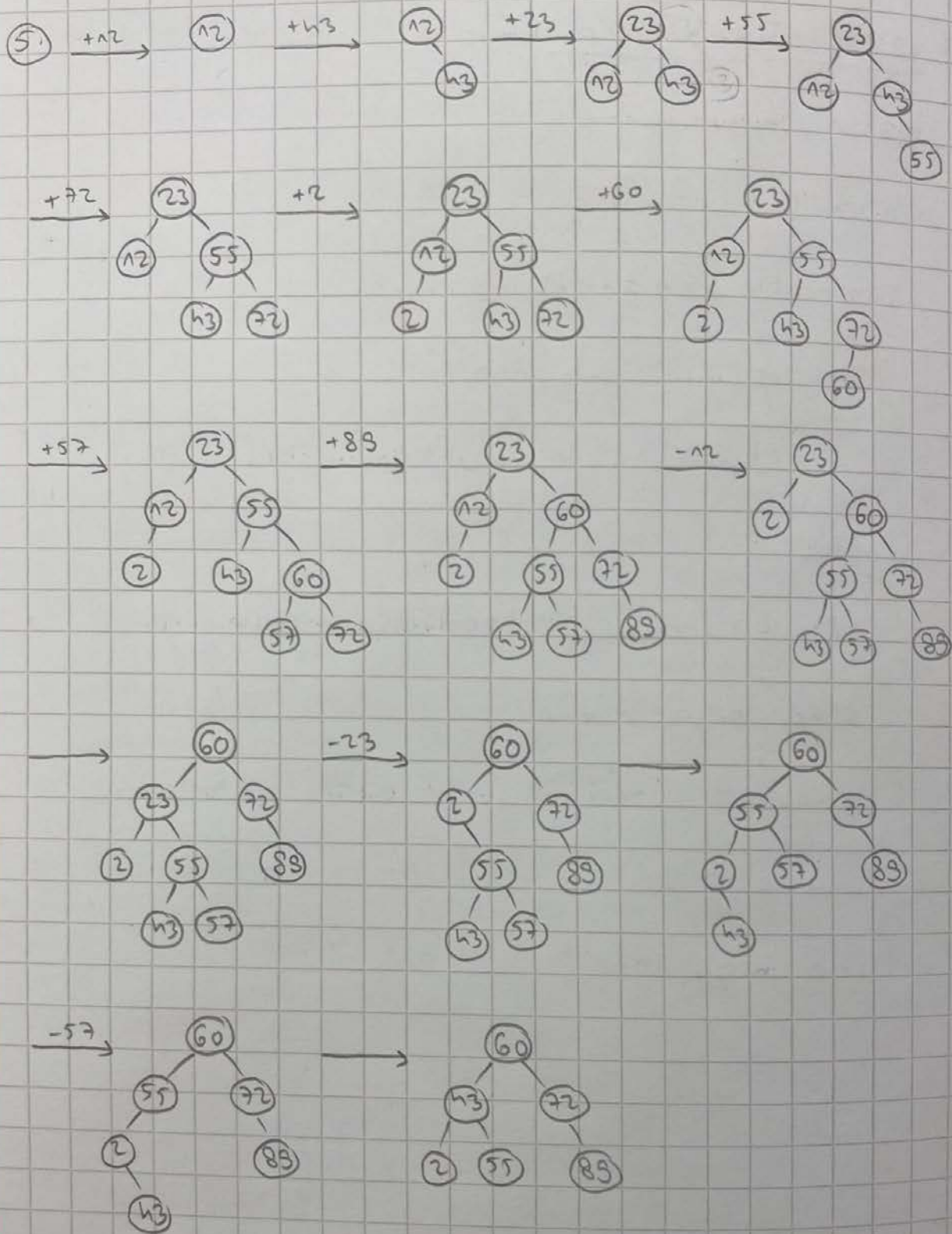
else high = mid - 1

end-if

end-while

return high, low

ACN K1-17



⑥ CHECK-AVL-BALANCED(bst)

$p = \text{root}(bst)$, $\text{QUEUE_INIT}(Q)$

$\text{INSERT}(Q, p)$

while (not $\text{QUEUE_EMPTY}(Q)$) do

$p = \text{DELETE}(Q)$

if ($\text{abs}(\text{hl}(p) - \text{hr}(p)) > 1$) then return false

end-if

if ($\text{left}(p) \neq \text{nil}$) then $\text{INSERT}(Q, \text{left}(p))$

end-if

if ($\text{right}(p) \neq \text{nil}$) then $\text{INSERT}(Q, \text{right}(p))$

end-if

end-while

return true

VLN-17

ACN

